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Uncertainty Evaluation of Residual Stress Measurement in Toughened Glass Using the Marshall–Lawn Indentation Model

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ABSTRACT

Uncertainty is an inherent component of any measured or model-derived quantity and depends on both the applied mathematical formulation and the uncertainties of its input parameters. Since direct measurement of compressive stress in glass is generally not feasible, mathematical models such as the Marshall–Lawn indentation formula are widely used. In this study, the uncertainty associated with this formula is evaluated following the ISO Guide 98 (GUM) framework. Sensitivity coefficients are obtained from the partial derivatives of the model, and a complete uncertainty budget is constructed based on experimental data and literature-derived inputs. The results provide a traceable and structured uncertainty analysis for estimating residual compressive stress in toughened glass surfaces.

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1. INTRODUCTION

Residual compressive stress is a key mechanical property that enhances the strength and durability of glass products. This stress is typically introduced through thermal or chemical tempering, improving resistance to crack propagation and mechanical failure (Marshall, D.B. & Lawn, B.R. (1979); Marshall, D.B., Lawn, B.R. & Chantikul, P. (1979)). Accurate and traceable measurement of surface stress is therefore essential for quality control, conformity assessment, and certification of toughened glass used in the construction, automotive, and electronics industries.

Optical methods such as birefringence are widely used because they are non-destructive and relatively simple to implement. However, these methods have important limitations. Absolute stress values cannot be measured

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directly, optical transparency is required, and the applicability of the technique depends strongly on sample geometry and size. In addition, birefringence captures stress differences only along the light path, often yielding averaged rather than surface-specific stress values (Marshall, D.B. & Lawn, B.R. (1979)).

Mechanical methods, such as strain gauges, face their own challenges, particularly in establishing a stress-free reference state on curved or treated surfaces. To overcome these limitations, Lawn and Marshall (Marshall, D.B., Lawn, B.R. & Chantikul, P. (1979)) proposed an indentation-based method that estimates surface residual stress from the geometry of cracks formed around Vickers indentations. This indentation-based method has gained wide acceptance due to its simplicity, accessibility, and suitability for various glass compositions and thicknesses (Marshall, D.B., Lawn, B.R. & Chantikul, P. (1979); Marshall, D. B., & Lawn, B. R. (1977)).

Despite its advantages, the indentation method remains an indirect technique. Residual stress is not measured directly; instead, it is calculated from applied load, crack dimensions, and material-dependent constants (Marshall, D.B., Lawn, B.R. & Chantikul, P. (1979); Marshall, D. B., & Lawn, B. R. (1977)). Each parameter carries its own uncertainty arising from instrument resolution, material variability, calibration, and model assumptions (Wiederhorn, S. M. (1969)). To ensure reliable results, these uncertainties must be systematically evaluated.

Despite the widespread use of indentation-based methods in ceramics and brittle materials research, uncertainty evaluation is often limited to experimental repeatability or statistical scatter, while comprehensive GUM-based uncertainty propagation analyses remain relatively limited. As a result, the metrological reliability and traceability of reported mechanical property measurements are not always explicitly addressed.

The Guide to the Expression of Uncertainty in Measurement (GUM) stipulates that both measured and model-derived quantities reported in conformity assessment must include a clear and traceable uncertainty statement (ISO Guide 98-1, (2024); Tavakoli Golpaygani, A., & Share Pasand, M. M. (2023); European cooperation for accreditation (EA). (2022); Eurachem, & Cooperation on International Traceability in Analytical Chemistry, (2012)). Laboratories accredited under ISO/IEC 17025 are likewise required to quantify and report measurement uncertainty (ISO/IEC 17025, (2017)).

Recent studies have emphasized both improved measurement techniques and rigorous uncertainty quantification. Lohr, Bukieda, and Weller investigated more than 80 samples of fully tempered and heat-strengthened glass, demonstrating measurable variability in both edge and surface stresses and linking stress variability to bending strength scatter (Lohr, K., Bukieda, P., & Weller, B. , (2020)). A 2024 analytical and experimental study clarified the stress–optic relationship and emphasized uncertainties arising from variations in the photoelastic constant, wavelength, and glass thickness, as well as modeling assumptions associated with stress distribution near edges (Efferz L., Thiele K., Schuster M., Schuler Ch., Siebert· G. & Schneider J. (2024)).

Large-format photoelastic characterization has recently advanced with the introduction of scattered-light polariscopes for panels up to 3 meters in length. A 2025 study demonstrated the industrial relevance of this approach but highlighted uncertainty challenges in transferring acceptance criteria from small standardized specimens to large architectural panes, as well as the need to correct for spatial stress inhomogeneity (Efferz L., Schuler Ch. & Siebert· G. (2025)). Process-modeling studies based on thermoelastic reconstruction using recorded thermal histories have further demonstrated that modeling assumptions and process-data uncertainty can contribute significantly to the overall error in residual stress estimation (Aronen A. (2024)).

In this study, we present a systematic uncertainty analysis of the Marshall–Lawn formula for calculating residual compressive stress in tempered glass. The methodology follows the GUM framework [8–10], applies sensitivity coefficients derived from partial derivatives of the model, and quantifies each contributing factor using experimental data, literature values, and expert judgment. The resulting uncertainty budget provides a practical reference for calibration laboratories, quality control engineers, and researchers working with tempered glass and related brittle materials.

The present work addresses the need for a systematic and traceable uncertainty evaluation of indentation-based residual stress measurements. Using the Marshall–Lawn model as a representative and widely adopted framework, the study applies GUM-based uncertainty propagation, sensitivity analysis, and uncertainty budgeting to identify the dominant sources of measurement uncertainty. The primary objective is not the development of a new residual stress equation but the establishment of a metrologically consistent framework that supports reliable interpretation, comparison, and practical application of residual stress measurements in tempered glass and related brittle materials.

2. The GUM method

The Guide to the Expression of Uncertainty in Measurement, published by the International Organization for Standardization (ISO), provides a unified method for the calculation, evaluation, and presentation of measurement uncertainties for conformity assessment bodies, including testing and calibration laboratories (ISO Guide 98-1, (2024); Share Pasand, M.M., Tavakoli Golpaygani, A., Ahmadi, S. & Nouri Kamari, M. (2024)). This section outlines the

GUM-based framework adopted for uncertainty propagation in the Marshall–Lawn residual stress model [9]. The following equations are presented in a stepwise manner to ensure transparency of the uncertainty propagation procedure and to maintain consistency with the GUM-based analytical framework adopted in this study. Consider the following mathematical model for a measurement:

$$y = f(x_1, x_2, \dots x_N) \quad (1)$$

where y represents the measurand, i.e., the quantity being measured or estimated and $x_1, x_2, \dots x_N \in R$ are the N quantities related to the measurement result. The term $f(.): R^N \to R$ is a known scalar-output, a differentiable multi-input function that admits a Taylor expansion (Share Pasand, M. M. & Tavakoli Golpaygani, A., (2023)).

Eq. (1) constitutes an algebraic, explicit and deterministic mathematical model of the measurement. Taking the expectation of both sides of Eq. (1) yields:

$$\mu_y = E\{f(x_1, x_2, \dots x_N)\} \quad (2)$$

where $E\{.\}$ represents the expected value with respect to the joint probability distribution function. The probability distribution is commonly assumed to be Gaussian (normal). However, with the exception of coverage factor and confidence interval computations, the rest of the calculations of for measurement uncertainty are independent of the probability distribution function (Share Pasand, M. M. & Tavakoli Golpaygani, A., (2023)). The deviation of the measurand from its average is:

$$y - \mu_y = f(x_1, x_2, \dots x_N) - \mu_y \quad (3)$$

The deviation in (3) is assumed to be sufficiently small, allowing first-order Taylor expansion of the function $f(.)$ as follows:

$$y - \mu_y \approx \sum_{i=1}^{N} \frac{\partial f}{\partial x_i}|_{\mu_{x_1},\dots\mu_{x_N}} (x_i - \mu_{x_i}) \quad (4)$$

where $\partial f/\partial x_i$ is the partial derivative of the function $f(.)$ that should be evaluated by substituting the expected values of $x_1, x_2, \dots x_N$ after derivation. Eq. (4) yields:

$$(y - \mu_y)^2 \approx \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 (x_i - \mu_{x_i})^2 + \cdots$$
$$2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}(x_i - \mu_{x_i})(x_j - \mu_{x_j}) \quad (5)$$

Using the definition of standard deviation, variance, and correlation coefficient in (5) leads to the following:

$$u_y^2 \approx \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 + \cdots$$
$$2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} r(x_i, x_j) u_{x_i} u_{x_j} \quad (6)$$

where $u_{x_1}, \dots u_{x_N}$ and u_y are standard deviations of contributing factors and the measurand, $r(x_i, x_j)$ is the correlation coefficient between x_i and x_j. If all contributing factors are mutually uncorrelated, (6) results the following:

$$u_y^2 \approx \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 \quad (7)$$

Eqs. (6) and (7) are known as uncertainty propagation equations and are widely used in uncertainty evaluation of measurements (European cooperation for accreditation (EA). (2022); Eurachem, & Cooperation on International Traceability in Analytical Chemistry, (2012)). The expanded uncertainty (ISO Guide 98-1, (2024)) is the standard deviation of the measurand σ_y multiplied by a coverage factor k. The coverage factor for Gaussian distribution with confidence level 95% interval is $k = 2$ (Tavakoli Golpaygani, A., & Share Pasand, M. M. (2023)).

Eqs. (6) and (7) state that the variance of a measurand can be approximately modeled as a weighted sum of variances of all contributing factors. This implies that each contributing factor affects the eventual uncertainty with a sensitivity coefficient equal to the magnitude of the partial derivative of $f(.)$ with respect to that contributing factor.

3. Uncertainty evaluation of residual stress in tempered glass surfaces

The uncertainty evaluation was performed according to the GUM framework using the Marshall–Lawn indentation model for residual stress estimation. The following equations are presented to summarize the analytical relations required for uncertainty propagation and sensitivity analysis.

The deformation pattern associated with Vickers indentation is schematized in Figure 1. Vickers indentation produces median cracks with an approximately half-penny geometry. The crack dimensions, denoted by C and C', increase with increasing indentation load P. These crack lengths are then used to estimate residual stress by comparison with untreated specimens.

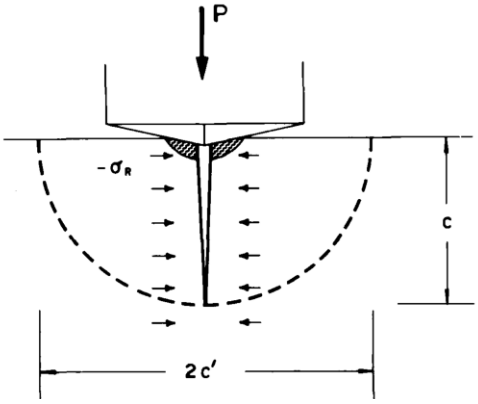

Figure 1. Schematic fracture pattern associated with Vickers indentation on tempered glass surface, showing median crack geometry (Marshall, D. B., & Lawn, B. R. (1977)).

$$K = \chi \frac{P}{C^{1.5}} - 2m\sigma_R \sqrt{\frac{C}{\pi}} \quad (8)$$

Assuming fracture occurs when $K = K_{IC}$, (i.e., the critical stress intensity factor (fracture toughness)). If we rearrange to solve for residual stress σ_R:

$$\sigma_R = \frac{1}{2m}\left[\frac{\chi P}{C^{1.5}} - K_{IC}\right]\left(\sqrt{\frac{\pi}{C}}\right) \quad (9)$$

where σ_R is the surface compressive residual stress, χ is the dimensionless indentation contact constant, C is the radial crack length, P is applied force in the tempered glass, and m is a modification factor whose value varies under different conditions. This factor is equal to unity when free-surface effects and stress gradients across the crack depth are neglected. K is the stress intensity factor, and K_{IC} is the critical stress intensity factor (fracture toughness), which is a material property experimentally determined from crack propagation tests such as double-cantilever beam or single-edge notched beam method. The model assumes that residual stress is uniformly distributed over the crack depth. Therefore, its effect appears as a function of $\sqrt{C}$. A linear plot of $\frac{P}{C^{1.5}}$ vs $\sqrt{C}$ would then enable σ_R for the tempered plate to be obtained from the slope.

Define the right-hand side of (9) as the function $f(.)$ given in (7). Taking partial derivatives from the right-hand side of (9) and substituting into (7) yields:

$$u_{\sigma_R}^2 \approx \left(\frac{\partial \sigma_R}{\partial K_{IC}}\right)^2 u_{K_C}^2 + \left(\frac{\partial \sigma_R}{\partial C}\right)^2 u_C^2 + \cdots \left(\frac{\partial \sigma_R}{\partial P}\right)^2 u_P^2 + \left(\frac{\partial \sigma_R}{\partial m}\right)^2 u_m^2 + \left(\frac{\partial \sigma_R}{\partial \chi}\right)^2 u_\chi^2 \quad (10)$$

which reads:

$$u_{\sigma_R}^2 \approx \left(\frac{-1}{2m}\left(\sqrt{\frac{\pi}{C}}\right)\right)^2 u_{K_C}^2 + \cdots \left(\frac{\sqrt{\pi}}{m}\left(-\frac{\chi P}{C^3} + \frac{K_{IC}}{4C^{1.5}}\right)\right)^2 u_C^2 + \cdots \left(\frac{1}{2m}\left[\frac{\chi}{C^{1.5}}\right]\left(\sqrt{\frac{\pi}{C}}\right)\right)^2 u_P^2 + \cdots \left(-\frac{\sigma_R}{m}\right)^2 u_m^2 + \left(\frac{P}{2mC^{1.5}}\left(\sqrt{\frac{\pi}{C}}\right)\right)^2 u_\chi^2 \quad (11)$$

where:

$$\frac{\partial \sigma_R}{\partial K_{IC}} = \frac{-1}{2m}\left(\sqrt{\frac{\pi}{C}}\right), \frac{\partial \sigma_R}{\partial C} = \frac{\sqrt{\pi}}{m}\left(-\frac{\chi P}{C^3} + \frac{K_{IC}}{4C^{1.5}}\right) \quad \frac{\partial \sigma_R}{\partial P} = \frac{\chi\sqrt{\pi}}{2mC^2}, \quad \frac{\partial \sigma_R}{\partial m} = -\frac{\sigma_R}{m} \quad \frac{\partial \sigma_R}{\partial \chi} = \frac{P}{2mC^{1.5}}\left(\sqrt{\frac{\pi}{C}}\right) \quad (12)$$

The contributing factors to determine the uncertainty in σ_R are summarized in Table 1. Moreover, Table 2 lists the statistical features of the contributing factors.

Table 1. Contributing factors to the uncertainty evaluation of residual stress using the Marshall–Lawn formula (Eq. 9).

Contributing factor	Symbol	Measurement method	Source of uncertainty	Reference
Critical stress intensity factor	K_{IC}	crack propagation tests	Material inhomogeneity, specimen geometry, loading rate	(Wiederhorn, S. M. (1969))
Radial crack length	C	Visual/ optical microscopy-SEM, image analysis	Optical measurement resolution, Operator variation, crack visibility	(Lawn, B. R., & Wilshaw, T. R. (1975))
Applied load	P	Indentation machine	Load cell calibration, dynamic effects, instrument drift	(ASTM E384-17. (2017))
Geometric correction factor	m	Empirical calibration, theoretical determination	Material and crack geometry, experimental conditions. Typically assumed to be 1, but subject to variation.	(Marshall, D. B., & Lawn, B. R. (1977))
Indentation contact constant	χ	Empirical calibration, theoretical determination	Indenter tip geometry, contact area calibration, material behavior	(Evans, A. G., & Charles, E. A. (1976))
Residual surface stress	σ_R	calculation	Calculated from other parameters using equation	-

Table 2. Statistical features of contributing factors to the uncertainty evaluation of residual stress using the Marshall–Lawn formula (Eq. 9).

Contributing factor	Symbol	Source of uncertainty	Evaluation type (A/B)	Probability density function	Typical Uncertainty
Critical stress intensity factor	K_{IC}	Random variations systematic errors, instrumental	A	Normal	±5% to ±10% Depending on the method used and the materials homogeneity
Radial crack length	C	Random variations systematic errors, instrumental	A	Uniform	±5% to ±15% Measuring from the surface or profile can affect accuracy. Using image analysis or SEM can improve precision
Applied load	P	systematic errors, instrumental	B	Uniform	±1% to ±5% Modern indenters have higher precision. Dead weight machines might introduce more error
Geometric correction factor	m	Model assumption, systematic	B	Normal	±5% to ±10% Less variability in thermally tempered glass with relatively uniform stress fields. It could be more for chemically tempered glasses
Indentation contact constant	χ	Systematic errors, instrumental	B	Normal	±10% to ±20% Depending on indenter sharpness, indenter calibration, and elastic/plastic zone behavior
Residual surface stress	σ_R	Combined uncertainty	B		±20% to ±30% This represents the combined uncertainty resulting from the propagation of uncertainties associated with P, C,χ and m

4. Model limitations and assumptions

The uncertainty analysis presented in this work applies the Marshall–Lawn equation (Eq. 9) under several simplifying assumptions that merit explicit clarification.

(1) Uniform stress distribution: The model assumes that the surface residual stress is uniform throughout the crack depth. In practice, tempered glasses often exhibit stress gradients and local inhomogeneities arising from variations in chemical composition, cooling rate, and tempering conditions.

(2) Idealized crack geometry: The indentation crack system is assumed to follow a half-penny geometry. However, the actual crack morphology may deviate from this idealized shape due to surface roughness, polishing damage, or compositional heterogeneity, which can influence both crack length and stress estimation.

(3) Independence of input parameters: The uncertainty propagation was initially carried out under the assumption that all input variables are statistically independent. However, partial correlations may exist, particularly between crack length and the empirical constants m and χ, since both are influenced by indentation geometry.

It is noted that the assumption of zero correlation among input quantities may affect the combined uncertainty. In particular, moderate correlations between crack length, the geometric correction factor, and the indentation contact constant may alter uncertainty propagation. According to the GUM framework, positive correlation coefficients (e.g., r = +0.5) tend to increase the combined standard uncertainty due to additional covariance terms, whereas negative correlations (e.g., r = −0.5) may partially reduce the overall uncertainty. However, rigorous quantification of these effects requires dedicated repeated measurements to estimate covariance matrices, which is beyond the scope of the present study. Therefore, input quantities were treated as uncorrelated in the baseline analysis.

To address these limitations, two complementary analyses are recommended: (a) performing a correlation analysis based on repeated indentation measurements to estimate covariance between key parameters; and (b) conducting a sensitivity analysis in which different assumed correlation coefficients (e.g., r = −0.5, 0, +0.5) are introduced to evaluate their influence on the combined uncertainty of σ_R. Incorporating these considerations would enhance the robustness of the uncertainty budget, clarify the validity range of the Marshall–Lawn approach, and improve the traceability of residual stress measurements in non-crystalline materials.

5. Case study: application of the Marshall–Lawn formula with experimental data

A representative experimental dataset reported in the literature was used as a benchmark case to demonstrate the implementation of the proposed uncertainty evaluation methodology and to assess the sensitivity of the Marshall–Lawn model to input uncertainties (Marshall, D. B., & Lawn, B. R. (1977)). Their indentation fracture studies provided calibrated constants that can be directly applied to the residual stress model. The nominal values of the input quantities, along with the statistical characteristics of the contributing factors, are summarized in Table 3. These values are representative of data reported in the foundational work of Marshall and Lawn (Marshall, D.B. & Lawn, B.R. (1979)). For parameters not explicitly quantified in the referenced case study, representative uncertainty ranges were adopted from typical values reported in the relevant indentation and glass mechanics literature for the purpose of uncertainty propagation analysis.

The uncertainty budget is presented in a qualitative form based on sensitivity analysis and relative standard uncertainties of input quantities. This classification (low, medium, high) reflects the relative influence of each parameter on the combined uncertainty obtained from the propagation law. A full numerical decomposition of percentage contributions was not included, as the primary objective of this work is the development of a GUM-consistent uncertainty framework rather than a detailed variance attribution model.

Table 3. Uncertainty budget for residual stress calculation according to (Marshall, D.B. & Lawn, B.R. (1979)).

Contributing factor	Symbol	Nominal value	unit	Standard uncertainty (Relative)	Contribution to σ_R uncertainty
Fracture toughness	K_{IC}	0.75±0.05	MPa·m^$^{1/2}$	±7%	Low
Radial crack length	C	110±10	μm	±15%	High
Applied load	P	90±2	N	±2%	Negligible
Indentation contact constant	χ	0.026±0.003	-	±12%	Medium
Geometric correction factor	m	1±0.1	-	±10%	High

5.1. Results of Uncertainty Evaluation

Using the nominal values from Table 3 in Eq. (9), the residual stress is calculated to be 129 MPa. The objective of the following analysis is to determine the combined standard uncertainty associated with this value. This value corresponds to the compressive surface stress introduced by tempering and is in excellent agreement with the ~128 MPa obtained by Marshall and Lawn using independent optical interferometry methods.

Propagation of uncertainties through the formula yields a combined relative standard uncertainty of approximately 22–28%. With a coverage factor of k = 2, the expanded uncertainty is:

σ_R =129±30 MPa (confidence level 95%)

This case study illustrates how the Marshall–Lawn formula, when combined with a rigorous uncertainty evaluation, can provide traceable and credible values for the compressive surface stress of tempered glass. Unlike nearly stress-free annealed samples (σ_R ≈ 0), tempered glass exhibits a residual compressive stress of about 130 MPa. The agreement with interferometric measurements reported by Marshall and Lawn (Marshall, D.B. & Lawn, B.R. (1979)) supports the practical applicability of the model. The dominant influence of crack length and geometric calibration constants underscores the importance of accurate crack measurement and careful calibration when applying the indentation method in practice.

Although this level of uncertainty may appear large, it is consistent with the indirect nature of the method. The dominant contributions arise from crack length measurement and empirical calibration constants, both of which inherently introduce variability. Thus, a 20–30% range is realistic for this type of indirect technique and is generally acceptable for process-monitoring applications, particularly since the results are in close agreement with interferometric measurements reported by Marshall and Lawn (Marshall, D.B. & Lawn, B.R. (1979)).

4. CONCLUSION(S)

In this study, a GUM-based uncertainty evaluation of residual compressive stress in tempered glass using the Marshall–Lawn indentation model was presented. The combined standard uncertainty was found to be approximately 20–30%, which is typical for this type of indirect measurement.

The analysis showed that crack length and empirical parameters, particularly the indentation contact constant and geometric correction factor, are the dominant contributors to the overall uncertainty, highlighting the importance of precise crack measurement and proper parameter calibration.

The results indicate that, despite inherent limitations, the Marshall–Lawn approach can provide a traceable and practically useful framework for residual stress estimation in brittle materials, consistent with ISO/IEC 17025 requirements. Extension of the methodology to chemically strengthened glass may require additional considerations due to highly non-uniform stress profiles and increased sensitivity to crack geometry and empirical parameters. The contribution of this study is not the development of a new residual stress equation, but the establishment of a structured and traceable uncertainty evaluation framework for an existing and widely used measurement methodology. Such analyses are essential for the reliable interpretation, comparison, and practical use of residual stress measurements in brittle materials and glass engineering.

From an industrial perspective, the reported uncertainty is generally acceptable for process monitoring and quality control, while for strict conformity assessment applications, appropriate decision rules and guard bands (e.g., ISO 14253-1) should be applied. Overall, the study improves the reliability, traceability, and comparability of residual stress measurements in tempered glass.

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